



Office of Curriculum, Instruction & Professional Development
INSTRUCTIONAL MATERIALS WORKSHOP

**Developed by the
Math Curriculum Office
Spring 2003**

**Mathematics
Content Standards Posters
For the Classroom**

PRECALCULUS / TRIG

Grades 9 – 12

Mathematical Analysis

This discipline combines many of the trigonometric, geometric, and algebraic techniques needed to prepare students for the study of calculus and strengthens their conceptual understanding of problems and mathematical reasoning in solving problems. Mathematical analysis is often combined with a course in trigonometry or perhaps with one in linear algebra to make a year-long precalculus course.

- 1.0 Students are familiar with, and can apply, polar coordinates and vectors in the plane. In particular, they can translate between polar and rectangular coordinates and can interpret polar coordinates and vectors graphically.
- 2.0 Students are adept at the arithmetic of complex numbers. They can use the trigonometric form of complex numbers and understand that a function of a complex variable can be viewed as a function of two real variables. They know the proof of DeMoivre's theorem.
- 3.0 Students can give proofs of various formulas by using the technique of mathematical induction.
- 4.0 Students know the statement of, and can apply, the fundamental theorem of algebra.
- 5.0 Students are familiar with conic sections, both analytically and geometrically:
 - 5.1 Students can take a quadratic equation in two variables; put it in standard form by completing the square and using rotations and translations, if necessary; determine what type of conic section the equation represents; and determine its geometric components (foci, asymptotes, and so forth).
 - 5.2 Students can take a geometric description of a conic section - for example, the locus of points whose sum of its distances from $(1, 0)$ and $(-1, 0)$ is 6, and derive a quadratic equation representing it.
- 6.0 Students find the roots and poles of a rational function and can graph the function and locate its asymptotes.
- 7.0 Students demonstrate an understanding of functions and equations defined parametrically and can graph them.
- 8.0 Students are familiar with the notion of the limit of a sequence and the limit of a function as the independent variable approaches a number or infinity. They determine whether certain sequences converge or diverge.

Linear Algebra

The general goal in this discipline is for students to learn the techniques of matrix manipulation so that they can solve systems of linear equations in any number of variables. Linear algebra is most often combined with another subject, such as trigonometry or mathematical analysis to make a year-long precalculus course.

- 1.0 Students solve linear equations in any number of variables by using Gauss-Jordan elimination.
- 2.0 Students interpret linear systems as coefficient matrices and the Gauss-Jordan method as row operations on the coefficient matrix.
- 3.0 Students reduce rectangular matrices to row echelon form.
- 4.0 Students perform addition on matrices and vectors.
- 5.0 Students perform matrix multiplication and multiply vectors by matrices and by scalars.
- 6.0 Students demonstrate an understanding that linear systems are inconsistent (have no solutions), have exactly one solution, or have infinitely many solutions.
- 7.0 Students demonstrate an understanding of the geometric interpretation of vectors and vector addition (by means of parallelograms) in the plane and in three-dimensional space.
- 8.0 Students interpret geometrically the solution sets of systems of equations. For example, the solution set of a single linear equation in two variables is interpreted as a line in the plane, and the solution set of a two-by-two system is interpreted as the intersection of a pair of lines in the plane.
- 9.0 Students demonstrate an understanding of the notion of the inverse to a square matrix and apply that concept to solve systems of linear equations.
- 10.0 Students compute the determinants of 2×2 and 3×3 matrices and are familiar with their geometric interpretations as the area and volume of the parallelepipeds spanned by the images under the matrices of the standard basis vectors in two-dimensional and three-dimensional spaces.
- 11.0 Students know that a square matrix is invertible if, and only if, its determinant is nonzero. They can compute the inverse to 2×2 and 3×3 matrices using row reduction methods or Cramer's rule.
- 12.0 Students compute the scalar (dot) product of two vectors in n -dimensional space and know that perpendicular vectors have zero dot product.

Trigonometry

Trigonometry uses the techniques that students have previously learned from the study of algebra and geometry. The trigonometric functions studied are defined geometrically rather than in terms of algebraic equations. Facility with these functions as well as the ability to prove basic identities regarding them is especially important for students intending to study calculus, more advanced mathematics, physics and other sciences, and engineering in college.

- 1.0** Students understand the notion of angle and how to measure it, in both degrees and radians. They can convert between degrees and radians.
- 2.0** Students know the definition of sine and cosine as y - and x -coordinates of points on the unit circle and are familiar with the graphs of the sine and cosine functions.
- 3.0** Students know the identity $\cos^2(x) + \sin^2(x) = 1$:
 - 3.1** Students prove that this identity is equivalent to the Pythagorean theorem (i.e., students can prove this identity by using the Pythagorean theorem and, conversely, they can prove the Pythagorean theorem as a consequence of this identity).
 - 3.2** Students prove other trigonometric identities and simplify others by using the identity $\cos^2(x) + \sin^2(x) = 1$. For example, students use this identity to prove that $\sec^2(x) = \tan^2(x) + 1$.
- 4.0** Students graph functions of the form $f(t) = A \sin(Bt + C)$ or $f(t) = A \cos(Bt + C)$ and interpret A , B , and C in terms of amplitude, frequency, period, and phase shift.
- 5.0** Students know the definitions of the tangent and cotangent functions and can graph them.
- 6.0** Students know the definitions of the secant and cosecant functions and can graph them.
- 7.0** Students know that the tangent of the angle that a line makes with the x -axis is equal to the slope of the line.
- 8.0** Students know the definitions of the inverse trigonometric functions and can graph the functions.
- 9.0** Students compute, by hand, the values of the trigonometric functions and the inverse trigonometric functions at various standard points.

- 10.0** Students demonstrate an understanding of the addition formulas for sines and cosines and their proofs and can use those formulas to prove and/ or simplify other trigonometric identities.
- 11.0** Students demonstrate an understanding of half-angle and double-angle formulas for sines and cosines and can use those formulas to prove and/ or simplify other trigonometric identities.
- 12.0** Students use trigonometry to determine unknown sides or angles in right triangles.
- 13.0** Students know the law of sines and the law of cosines and apply those laws to solve problems.
- 14.0** Students determine the area of a triangle, given one angle and the two adjacent sides.
- 15.0** Students are familiar with polar coordinates. In particular, they can determine polar coordinates of a point given in rectangular coordinates and vice versa.
- 16.0** Students represent equations given in rectangular coordinates in terms of polar coordinates.
- 17.0** Students are familiar with complex numbers. They can represent a complex number in polar form and know how to multiply complex numbers in their polar form.
- 18.0** Students know DeMoivre's theorem and can give n th roots of a complex number given in polar form.
- 19.0** Students are adept at using trigonometry in a variety of applications and word problems.

1.0**Mathematical Analysis Content Standards**

Students are familiar with, and can apply, polar coordinates and vectors in the plane. In particular, they can translate between polar and rectangular coordinates and can interpret polar coordinates and vectors graphically.

2.0

Mathematical Analysis Content Standards

Students are adept at the arithmetic of complex numbers. They can use the trigonometric form of complex numbers and understand that a function of a complex variable can be viewed as a function of two real variables.

They know the proof of DeMoivre's theorem.

3.0

Mathematical Analysis Content Standards

Students can give proofs of various formulas by using the technique of mathematical induction.

4.0

Mathematical Analysis Content Standards

**Students know the statement of,
and can apply, The Fundamental
Theorem of Algebra.**

5.0

Mathematical Analysis Content Standards

Students are familiar with conic sections, both analytically and geometrically.

5.1

Mathematical Analysis Content Standards

Students can take a quadratic equation in two variables; put it in standard form by completing the square and using rotations and translations, if necessary; determine what type of conic section the equation represents; and determine its geometric components: foci, asymptotes, and so forth.

5.2

Mathematical Analysis Content Standards

Students can take a geometric description of a conic section - for example, the locus of points whose sum of its distances from $(1, 0)$ and $(-1, 0)$ is 6, and derive a quadratic equation representing it.

6.0

Mathematical Analysis Content Standards

Students find the roots and poles of a rational function and can graph the function and locate its asymptotes.

7.0

Mathematical Analysis Content Standards

Students demonstrate an understanding of functions and equations defined parametrically and can graph them.

8.0

Mathematical Analysis Content Standards

Students are familiar with the notion of the limit of a sequence and the limit of a function as the independent variable approaches a number or infinity. They determine whether certain sequences converge or diverge.

1.0

Linear Algebra Content Standards

Students solve linear equations in any number of variables by using Gauss-Jordan elimination.

2.0

Linear Algebra Content Standards

Students interpret linear systems as coefficient matrices and the Gauss-Jordan method as row operations on the coefficient matrix.

3.0

Linear Algebra Content Standards

Students reduce rectangular matrices to row echelon form.

4.0

Linear Algebra Content Standards

**Students perform addition on
matrices and vectors.**

5.0

Linear Algebra Content Standards

Students perform matrix multiplication and multiply vectors by matrices and by scalars.

6.0

Linear Algebra Content Standards

Students demonstrate an understanding that linear systems are inconsistent and have no solutions, have exactly one solution, or have infinitely many solutions.

7.0**Linear Algebra Content Standards**

Students demonstrate an understanding of the geometric interpretation of vectors and vector addition (by means of parallelograms) in the plane and in three-dimensional space.

8.0

Linear Algebra Content Standards

Students interpret geometrically the solution sets of systems of equations. For example, the solution set of a single linear equation in two variables is interpreted as a line in the plane, and the solution set of a two-by-two system is interpreted as the intersection of a pair of lines in the plane.

9.0

Linear Algebra Content Standards

Students demonstrate an understanding of the notion of the inverse to a square matrix and apply that concept to solve systems of linear equations.

10.0**Linear Algebra Content Standards**

Students compute the determinants of 2×2 and 3×3 matrices and are familiar with their geometric interpretations as the area and volume of the parallelepipeds spanned by the images under the matrices of the standard basis vectors in two-dimensional and three-dimensional spaces.

11.0

Linear Algebra Content Standards

Students know that a square matrix is invertible if, and only if, its determinant is nonzero.

They can compute the inverse to 2×2 and 3×3 matrices using row reduction methods or Cramer's rule.

12.0

Linear Algebra Content Standards

Students compute the scalar (dot) product of two vectors in n -dimensional space and know that perpendicular vectors have zero dot product.

1.0**Trigonometry Content Standards**

Students understand the notion of angle and how to measure it, in both degrees and radians.

They can convert between degrees and radians.

2.0

Trigonometry Content Standards

Students know the definition of sine and cosine as y -and x -coordinates of points on the unit circle and are familiar with the graphs of the sine and cosine functions.

3.0

Trigonometry Content Standards

Students know the identity

$$\cos^2(x) + \sin^2(x) = 1$$

3.1

Trigonometry Content Standards

Students prove that this identity is equivalent to the Pythagorean theorem (i.e., students can prove this identity by using the Pythagorean theorem and, conversely, they can prove the Pythagorean theorem as a consequence of this identity).

3.2

Trigonometry Content Standards

Students prove other trigonometric identities and simplify others by using the identity $\cos^2(x) + \sin^2(x) = 1$.

For example, students use this identity to prove that $\sec^2(x) = \tan^2(x) + 1$.

4.0**Trigonometry Content Standards**

Students graph functions of the form

$$f(t) = A \sin(Bt + C)$$

or $f(t) = A \cos(Bt + C)$

and interpret A , B , and C in terms of amplitude, frequency, period, and phase shift.

5.0

Trigonometry Content Standards

Students know the definitions of the tangent and cotangent functions and can graph them.

6.0

Students know the definitions of the secant and cosecant functions and can graph them.

7.0**Trigonometry Content Standards**

Students know that the tangent of the angle that a line makes with the x -axis is equal to the slope of the line.

8.0

Trigonometry Content Standards

Students know the definitions of the inverse trigonometric functions and can graph the functions.

9.0

Trigonometry Content Standards

Students compute, by hand, the values of the trigonometric functions and the inverse trigonometric functions at various standard points.

10.0

Trigonometry Content Standards

Students demonstrate an understanding of the addition formulas for sines and cosines and their proofs and can use those formulas to prove and/or simplify other trigonometric identities.

11.0

Trigonometry Content Standards

Students demonstrate an understanding of half-angle and double-angle formulas for sines and cosines and can use those formulas to prove and/or simplify other trigonometric identities.

12.0

Trigonometry Content Standards

Students use trigonometry to determine unknown sides or angles in right triangles.

13.0

Trigonometry Content Standards

Students know the law of sines and the law of cosines and apply those laws to solve problems.

14.0**Trigonometry Content Standards**

Students determine the area of a triangle, given one angle and the two adjacent sides.

$$\text{Area} = \frac{1}{2} bc \sin(A)$$

15.0**Trigonometry Content Standards**

Students are familiar with polar coordinates. In particular, they can determine polar coordinates of a point given in rectangular coordinates and vice versa.

$$\mathbf{x = r \text{ Cos}(\theta), \quad y = r \text{ Sin}(\theta)}$$

$$\mathbf{r = \sqrt{x^2 + y^2}, \quad \theta = \text{tan}^{-1}\left(\frac{y}{x}\right)}$$

16.0

Trigonometry Content Standards

Students represent equations given in rectangular coordinates in terms of polar coordinates.

17.0

Trigonometry Content Standards

Students are familiar with complex numbers. They can represent a complex number in polar form and know how to multiply complex numbers in their polar form.

18.0

Trigonometry Content Standards

Students know DeMoivre's Theorem and can give n th roots of a complex number given in polar form.

19.0

Trigonometry Content Standards

Students are adept at using trigonometry in a variety of applications and word problems.