### Unit Goals – Stage 1

**Number of Days:** 34 days  
2/27/17 – 4/13/17

**Unit Description:** Deriving new formulas from previously discovered ones, the students will leave Unit 5 with an understanding of area and volume formulas for polygons, circles and their three-dimensional counterparts. Unit 5 seeks to have the students develop and describe a process whereby they are able to solve for areas and volumes in both procedural and applied contexts. Units of measurement are emphasized as a method of checking one’s approach to a solution. Using their study of circles from Unit 3, the students will dissect the Pythagorean Theorem. Through this they will develop properties of the special right triangles: 45°, 45°, 90° and 30°, 60°, 90°. Algebra skills from previous classes lead students to understand the connection between the distance formula and the Pythagorean Theorem. In all instances, surface area will be generated using the smaller pieces from which a given shape is constructed. The student will, again, derive new understandings from old ones.

**Materials:** Construction tools, construction paper, patty paper, calculators, Desmos, rulers, glue sticks (optional), sets of geometric solids, pennies and dimes (Explore p. 458), string, protractors, paper clips, square, isometric and graph paper, sand or water, plastic dishpan

### Standards for Mathematical Practice

| SMP 1 | Make sense of problems and persevere in solving them. |
| SMP 2 | Reason abstractly and quantitatively. |
| SMP 3 | Construct viable arguments and critique the reasoning of others. |
| SMP 4 | Model with mathematics. |
| SMP 5 | Use appropriate tools strategically. |
| SMP 6 | Attend to precision. |
| SMP 7 | Look for and make use of structure. |
| SMP 8 | Look for and express regularity in repeated reasoning. |

### Transfer Goals

**Students will be able to independently use their learning to...**
- Make sense of never-before-seen problems and persevere in solving them.
- Construct viable arguments and critique the reasoning of others.

### Making Meaning

**UNDERSTANDINGS**

**Students will understand that...**

- By cutting apart and rearranging figures, new formulas can be derived from old ones.
- Units are a key to differentiating between area, \(u^2\), and volume, \(u^3\).
- The Pythagorean Theorem leads to the distance formula and the equation of a circle.
- In a right triangle, the area of the square on the hypotenuse is the sum of the areas of the squares on the legs.
- \(\pi\) is the constant of proportionality between the circumference and the diameter of a circle.
- If a shape is divided into subregions, the sum of the areas of those subregions, even if they are rearranged, is equal to the area of the original shape.
- When a triangle or parallelogram is rotated so that a different side is its base, the area of that triangle or parallelogram holds constant.
- To find the surface area of a prism, pyramid, cylinder or cone, combine the shape’s lateral surface area with the area(s) of the shape’s base(s).
- Sub-shapes or near-shapes can be used to approximate surface area and volume.

### Standards for Mathematical Content Clusters Addressed

| m G-SRT.C | Define trigonometric ratios and solve problems involving right triangles. |
| a G-C.B | Find arc lengths and areas of sectors of circles. |

### ESSENTIAL QUESTIONS

**Students will keep considering...**

- Do we need to memorize new formulas or can we use old formulas in a new way?
- If we have a shape with a given area, and we cut that shape apart and rearrange its pieces, will the area remain constant?
- How are the area formulas for a square, a rectangle, a trapezoid, and a triangle related?
- Why do the solution points for \((x - h)^2 + (y - k)^2 = r^2\) form a circle?
- How can you find the volume of a solid for which no formula is available?
- How can a concept map or a Venn diagram be constructed to organize the solids based on their commonalities?
- How is finding the volume of a prism similar to finding the area of a rectangle?
<table>
<thead>
<tr>
<th>GPE.A</th>
<th>Translate between the geometric description and the equation for a conic section.</th>
</tr>
</thead>
<tbody>
<tr>
<td>GPE.B</td>
<td>Use coordinates to prove simple geometric theorems algebraically.</td>
</tr>
<tr>
<td>GMD.A</td>
<td>Explain volume formulas and use them to solve problems.</td>
</tr>
<tr>
<td>MG.A</td>
<td>Apply geometric concepts in modeling situations.</td>
</tr>
</tbody>
</table>

**KNOWLEDGE**

*Students will know…*

- Formulas and methods for finding the areas of rectangles, parallelograms, triangles, trapezoids, kites, regular polygons, circles, sectors, segments, and annuluses.
- The Pythagorean Theorem and that, given \( a^2 + b^2 = c^2 \), \( a \) and \( b \) are the lengths of the legs. The length of the hypotenuse is \( c \).
- The Converse of the Pythagorean Theorem.
- If two figures are similar with a scale factor \( s \), then their areas have scale factor \( s^2 \).
- The area of a sector is the same part of the area of the whole circle as the measure of the central angle of the sector is of 360°.
- In a 45°-45°-90° triangle, if the legs have length \( m \), then the hypotenuse has length \( m \sqrt{2} \). In a 30°-60°-90° triangle, if the leg opposite the 30° has length \( a \), then the hypotenuse has length \( 2a \) and the other leg as length \( a \sqrt{3} \).
- The equation \( (x - h)^2 + (y - k)^2 = r^2 \) describes a circle with points on the circle \((x, y)\), a radial distance of \( r \), and center \((h, k)\).
- The volume formulas for prisms, cylinders, pyramids, cones, and spheres.

**SKILLS**

*Students will be skilled at and/or be able to…*

- Derive formulas for the areas of rectangles, parallelograms, triangles, trapezoids, kites, regular polygons, circles, annuluses, sectors, segments of circles, and surface areas of spheres.
- Derive the fact that the length of the arc intercepted by an angle is proportional to the radius.
- Apply surface area formulas to solve problems.
- Solve area application problems using various problem-solving strategies.
- Solve for the missing side length of a triangle using the Pythagorean Theorem.
- Use the Converse of the Pythagorean Theorem to prove that a triangle is right.
- Solve for the lengths of the sides of a 45°-45°-90° triangle and a 30°-60°-90° triangle.
- Derive the distance formula from the Pythagorean Theorem.
- Use the distance formula to solve problems.
- Derive the equation of a circle of given center and radius using the Pythagorean Theorem.
- Apply the Pythagorean relationship to problems involving circles.
- Use coordinates to compute perimeters of polygons and areas of triangles and rectangles.
- Derive formulas for finding the volumes of prisms, cylinders, pyramids, cones, and spheres using dissection arguments and Cavalieri’s Principle.
- Solve applied problems involving polyhedra, cones, cylinders, spheres, or hemispheres.
- Use displacement to find the volumes of irregularly shaped solids.
## Standards for Mathematical Practice

| SMP 1 | Make sense of problems and persevere in solving them. |
| SMP 2 | Reason abstractly and quantitatively. |
| SMP 3 | Construct viable arguments and critique the reasoning of others. |
| SMP 4 | Model with mathematics. |
| SMP 5 | Use appropriate tools strategically. |
| SMP 6 | Attend to precision. |
| SMP 7 | Look for and make use of structure. |
| SMP 8 | Look for and express regularity in repeated reasoning. |

## Standards for Mathematical Content

<p>| | |</p>
<table>
<thead>
<tr>
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<tbody>
<tr>
<td><strong>[m]</strong> G-SRT.C</td>
<td>Define trigonometric ratios and solve problems involving right triangles.</td>
</tr>
<tr>
<td>G-SRT.6</td>
<td>Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.</td>
</tr>
<tr>
<td><strong>[a]</strong> G-C.B</td>
<td>Find arc lengths and areas of sectors of circles.</td>
</tr>
<tr>
<td>G-C.5</td>
<td>Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.</td>
</tr>
<tr>
<td><strong>[a]</strong> G-GPE.A</td>
<td>Translate between the geometric description and the equation for a conic section.</td>
</tr>
<tr>
<td>G-GPE.1</td>
<td>Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.</td>
</tr>
<tr>
<td><strong>[m]</strong> G-GPE.B</td>
<td>Use coordinates to prove simple geometric theorems algebraically.</td>
</tr>
<tr>
<td>G-GPE.7</td>
<td>Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.*</td>
</tr>
<tr>
<td><strong>[a]</strong> G-GMD.A</td>
<td>Explain volume formulas and use them to solve problems.</td>
</tr>
<tr>
<td>G-GMD.1</td>
<td>Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri’s Principle, and informal limit arguments.</td>
</tr>
<tr>
<td>G-GMD.2 (+)</td>
<td>Give an informal argument using Cavalieri’s principle for the formulas for the volume of a sphere and other solid figures.</td>
</tr>
<tr>
<td>G-GMD.3</td>
<td>Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.</td>
</tr>
<tr>
<td><strong>[m]</strong> G-MG.A</td>
<td>Apply geometric concepts in modeling situations.</td>
</tr>
<tr>
<td>G-MG.1</td>
<td>Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder). ★</td>
</tr>
<tr>
<td>G-MG.2</td>
<td>Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot). ★</td>
</tr>
</tbody>
</table>

### Key

- **[m]** = major clusters; **[s]** = supporting clusters, **[a]** = additional clusters
- * Indicates a modeling standard linking mathematics to everyday life, work, and decision-making
- CA Indicates a California-only standard
Unit Assessment

Claim 1: Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency.

Concepts and skills that may be assessed in Claim 1:

[m] G-SRT.C
• Students will use similarity to show that side ratios in right triangles are properties of the angles in the triangle.

[a] G-C.B
• Students will, using similarity, derive the fact that the length of the arc intercepted by an angle is proportional to the radius.
• Students will derive the formula for the area of a sector.

[a] G-GPE.A
• Students will use the Pythagorean Theorem to derive the equation of a circle when given its center and radius.

[m] G-GPE.B
• Students will use coordinates to compute the perimeters of polygons.
• Students will use coordinates to compute the area of triangles and rectangles.

[a] G-GMD.A
• Students will give an informal argument for the area of a circle.
• Students will give an informal argument for the volume of a cylinder, pyramid and cone.
• Students will use dissection arguments and Cavalieri’s Principle.
• Students will use formulas to solve volume problems involving cylinders, pyramids, cones and spheres.

[m] G-MG.A
• Students will use geometric shapes, their measures, and their properties to describe objects.
• Students will apply concepts of density based on area and volume in modeling situations.

Claim 2: Students can solve a range of well-posed problems in pure and applied mathematics, making productive use of knowledge and problem-solving strategies.

Standard clusters that may be assessed in Claim 2:
• G-SRT.C

Claim 3: The student can clearly and precisely construct viable arguments to support their own reasoning and critique the reasoning of others.

Standard clusters that may be assessed in Claim 3:
• None

Claim 4: The student can analyze complex, real-world scenarios and can construct and use mathematical models to interpret and solve problems.

Standard clusters that may be assessed in Claim 4:
• G-MGD.A
• G-MG.A
Formative Assessment Opportunities

- Opening Tasks
- Informal teacher observations
- Checking for understanding using active participation strategies
- Exit slips/Summaries
- Modeling Lessons (SMP 4)
- Tasks
- Formative Assessment Lessons (FAL)
- Quizzes/Chapter Tests
- SBAC Interim Assessment Blocks

<table>
<thead>
<tr>
<th>Days</th>
<th>Learning Target</th>
<th>Expectations</th>
<th>Discovering Geometry (Activities and Lessons)</th>
<th>Curriculum Intranet</th>
<th>Application:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td><strong>Opening Task - The Open-Topped Box</strong></td>
<td></td>
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<td><strong>The Open-Topped Box</strong></td>
</tr>
<tr>
<td>1-2 days</td>
<td>I will explore the relationship between the side length and volume of a rectangular box.</td>
<td>To explore the relationship between length and volume, students design and construct an open-topped box. Once the box is complete, they solve for surface area and volume. After gathering the data onto a scatter plot ($x =$ corner edge length; $y =$ volume), students use a graphing program to solve for a regression line. Using that line, students can discuss data trends, domain and range, and decide which of the boxes has the greatest volume based on the height of the box. The Supplemental Resources on the Curriculum Intranet provide videos of students in the process of a similar activity, an article with teacher suggestions for conducting this activity, and instructions on creating scatterplots and regression lines with the TI-84. This activity serves to motivate and review topics that will be used throughout Unit 5.</td>
<td></td>
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</tr>
</tbody>
</table>
| 4-5 days | I will investigate the area of polygons by…                                     | • Deriving formulas for areas of rectangles, parallelograms, triangles, trapezoids and regular polygons.  
• Rearranging figures to derive new formulas from previously discovered formulas.  
• Solving area problems which require several steps.  
• Describing the process used when solving area problems.  
• Answering questions such as…  
  o What variables have you seen used in area formulas for rectangles? What do those variables represent?  
  o How do you know which side is the length and which side is the width?  
  o How are rectangles and parallelograms the same? Different?  
  o How can the area formula for a rectangle be the same as the area formula for a parallelogram?  
  o Is there more than one way that a parallelogram can be cut up and rearranged into a rectangle? | • Lesson 8-1  
• Lesson 8-2  
• Lesson 8-3  
• Lesson 8-4 |                                         | **Conceptual Understanding:**  
• Illuminations: Area Formulas                                                                 |
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| 3-4 days | I will solve problems involving circles by… | • Deriving new formulas from old to solve for the area of a: circle, sector, annulus, and segment.  
• Applying new formulas to solve multi-step problems.  
• Applying area formulas to solve for geometric probabilities.  
• Finding the surface areas of solids including cones and cylinders.  
• Answering questions such as…  
  o What are ways to remember that area is \( \pi r^2 \) and circumference is \( 2\pi r \) ?  
  o Why is \( \pi \) in these formulas?  
  o How long do we carry \( \pi \) along in our solutions rather than rounding to decimals? Why?  
  o When would we want to express our answers using the symbol \( \pi \) and when would we want to use a decimal equivalent?  
  o Given a “slice of pizza”, how can you determine what part the “slice” is of the entire area of the pizza? | • Lesson 8-5  
• Lesson 8-6  
• Lesson 8-7  
• Explore p. 458 | Conceptual Understanding:  
• Open Middle: Which Circle Is Bigger?  
Application:  
• Illuminations: Gerrymandering – Is It or Isn’t It  
• 3-Act Lesson: Rotunda West Florida  
• 3-Act Lesson: Holy Cow  
• MathVision Project: Pied |
### Unit 5 Area, the Pythagorean Theorem, and Volume

**Geometry ACC**

**Learning Plan – Stage 3**

**Suggested Sequence of Key Learning Events and Instruction**

<table>
<thead>
<tr>
<th>Days</th>
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</table>
| 2-3 days | I will investigate the Pythagorean Theorem and its converse by… | • Dissecting the squares on the legs of a right triangle and fitting the pieces into the square on the hypotenuse to prove the Pythagorean Theorem.  
• Applying the Pythagorean Theorem to find missing lengths in right triangles.  
• Using the Converse of the Pythagorean Theorem to determine right triangles.  
• Answering questions such as…  
  o Why is the longest leg always opposite the right angle in a right triangle?  
  o What is the difference between a theorem and a conjecture?  
  o Does the Pythagorean Theorem work for ALL triangles?  
  o If the sides of a triangle satisfy $a^2 + b^2 = c^2$, what can you say about the triangle? Remember, the converse of a true statement is not always true. | • Lesson 9-1  
• Lesson 9-2 | • MathVision Project: Madison’s Round Garden  
• Illustrative Mathematics: Setting Up Sprinklers  
• Conceptual Understanding:  
  • OpenMathRef: A Graphical Proof of the Pythagorean Theorem |
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</tr>
</thead>
</table>
| 2-3 days | I will apply the Pythagorean Theorem and its converse by… | - Exploring the special properties of special right triangles: 45°-45°-90° and 30°-60°-90°.  
- Solving multi-step real-world problems.  
- Answering questions such as…  
  o What do you always know about an isosceles right triangle?  
  o What do you always know about a right triangle with a leg that is half the length of the hypotenuse?  
  o Knowing the side length ratios of the special right triangles will be necessary in math class up through Precalculus and Calculus. What can you do to help you remember these side length ratios?  
  o What are steps that you can take to solve a word problem? | - Lesson 9-3  
- Lesson 9-4 | Procedural Skills and Fluency:  
- Geogebra Interactive: Special Right Triangles |
| 1-2 days | I will use coordinates in geometry by… | - Using coordinates to derive the distance formula.  
- Recognizing the connection between the distance formula and the Pythagorean Theorem.  
- Using the center and a point on the circumference to solve for the equation of a circle.  
- Solving problems using the distance formula.  
- Exploring rates of change via the Pythagorean Theorem.  
- Answering questions such as…  
  o How would you use a right triangle to find the distance between two points?  
  o If you are going to use a calculator to solve a distance problem, what are some things that you can do to be more likely to get a correct answer?  
  o How important is it to be exact with your substitution into the distance formula’s \((x_1, y_1)\) and \((x_2, y_2)\)? | - Lesson 9-5  
- Explore p. 507 | Conceptual Understanding:  
- Illustrative Mathematics: Triangle Perimeters  
Procedural Skills and Fluency:  
- MathOpenRef: Trapezoid (Coordinate Geometry) |
# Learning Plan – Stage 3

## Suggested Sequence of Key Learning Events and Instruction

<table>
<thead>
<tr>
<th>Days</th>
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</thead>
</table>
| 1-2 days | I will use the Pythagorean Theorem in circle geometry by... | • Solving multi-step problems.  
• Combining different conjectures to solve a variety of problems.  
• Answering questions such as...  
  o How did you combine conjectures, formulas and properties to solve problems in this section? | Lesson 9-6 | |
| 4-5 days | I will investigate the volumes of solids by... | • Identifying prisms, pyramids, cones and cylinders and describing their attributes.  
• Investigating relationships between the volumes of prisms and pyramids, and cylinders and cones.  
• Visualizing three-dimensional objects using models and nets.  
• Constructing informal arguments for the volumes of prisms, pyramids, cylinders, and cones.  
• Answering questions such as...  
  o What is the difference between a polyhedron and a polygon?  
  o How do you know how many faces a polyhedron has?  
  o Where are the base, vertex and altitude of a polyhedron?  
  o What is the difference between “right” and “oblique”?  
  o In what case is a cone a special case of a pyramid?  
  o Is it possible for a sphere to have two great circles that do not intersect?  
  o How did you combine conjectures, formulas and properties to solve problems in this section?  
  o How does the concept of volume differ from the concept of area?  
  o Just looking at the units, how can you tell if a solution is linear or volume or area?  
  o What would be a convincing example of the Oblique Prism-Cylinder Volume Conjecture?  
  o How does the volume of a pyramid or cone relate to the volume of its circumscribed prism or cylinder?  
  o Explain how you derived the volume formulas in this section. | Lesson 10-1  
Lesson 10-2  
Lesson 10-3  
Explore p. 544 | Conceptual Understanding:  
• Identify the Shapes of 2D cross-sections of 3D objects and identify 3D objects generated by rotations of 2D objects (video)  
• Illustrative Mathematics: Use Cavalieri’s Principle to Compare Aquarium Volumes  
Procedural Skills and Fluency:  
• MathOpenRef: Solid Geometry |
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|      |                | o The area of a triangle is ½ the area of the rectangle with the same base and height. Why, then, is the area of a cone or pyramid 1/3 that of its circumscribed cylinder or prism?  
o If the dimensions of the base of a solid are doubled and the height of that solid is doubled, by how much is the volume increased? | • Lesson 10-4  
• Lesson 10-5 | Conceptual Understanding:  
• Volume Displacement and Density Activity  
Application:  
• Illuminations: Cubed Cans  
• 3-Act Lesson: Water Tank  
• MathVision Project: Sand Castles  
• 3-Act Lesson: Fill ‘Er Up |
| 2-3 days | I will solve volume and displacement problems by… | • Combining understanding of area and the Pythagorean Theorem with knowledge of volume formulas.  
• Using displacement to calculate the volume and density of an irregularly shaped object.  
• Answering questions such as…  
o How could you find the volume of an irregularly shaped object?  
o What are steps that you can take to solve a word problem?  
o What is the difference between volume, mass, and density?  
o What weighs more, a pound of lead or a pound of feathers?  
o Will heavier objects displace more water than lighter objects of the same shape? | | |
| 2-3 days | I will investigate spheres by… | • Deriving the formula for the volume of a sphere.  
• Using the volume formulas for a sphere and a pyramid to derive the surface area formula for a sphere.  
• Applying volume and surface area formulas to problems involving spheres or hemispheres.  
• Answering questions such as…  
o Compare the volume of a sphere to the volume of a cylinder, cone or cube with the same radius and/or height. | • Lesson 10-6  
• Lesson 10-7 | Conceptual Understanding:  
• Volume Formulas (interactive)  
Procedural Skills and Fluency: |
## Learning Plan – Stage 3

### Suggested Sequence of Key Learning Events and Instruction

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</thead>
<tbody>
<tr>
<td>1-2 days</td>
<td>I will check my understanding of volume by participating in the FAL.</td>
<td><strong>Formative Assessment Lesson</strong> Volumes of Compound Objects</td>
<td></td>
<td>• MathOpenRef: Solid Geometry</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Application:</td>
<td>• Illuminations: Ice Cream Puddle</td>
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<td>• 3-Act Lesson: Meatballs</td>
</tr>
<tr>
<td>2 days</td>
<td>I will prepare for the unit assessment on area, the Pythagorean Theorem, and volume by…</td>
<td>Incorporating the Standards for Mathematical Practice (SMPs) along with the content standards to review the unit.</td>
<td></td>
<td>Conceptual Understanding:</td>
</tr>
<tr>
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<td></td>
<td></td>
<td></td>
<td>• FAL: Volumes of Compound Objects</td>
</tr>
<tr>
<td>1-2 days</td>
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<td></td>
<td>Procedural Skills and Fluency:</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td>• Geometry Review</td>
</tr>
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**Unit Assessment** (LBUSD Math Intranet, Assessment)